

# OPERATION CONCENTRATION

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# Outline

Who is Operation  
Concentration?

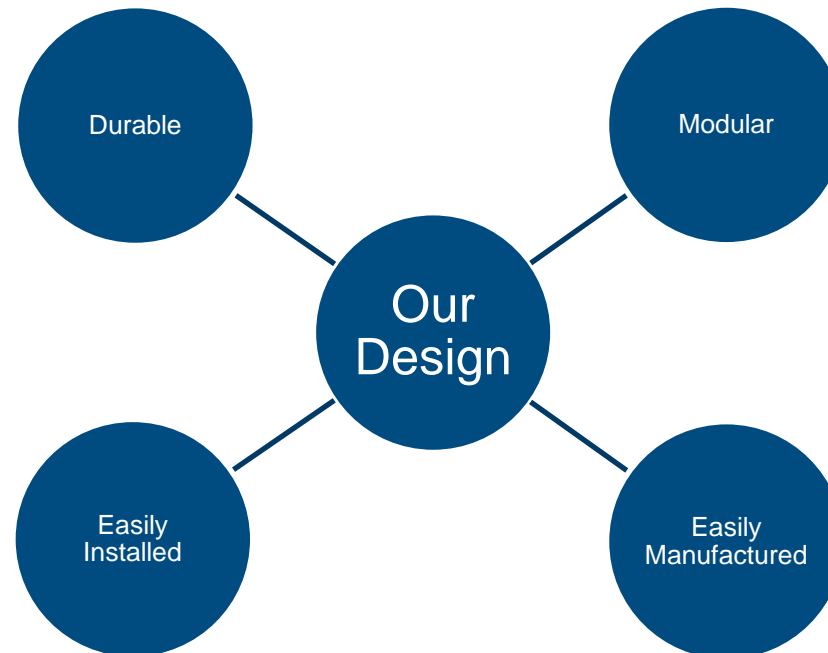
Overview of the design  
and its subsystems

Analysis of the  
subsystems

Final remarks

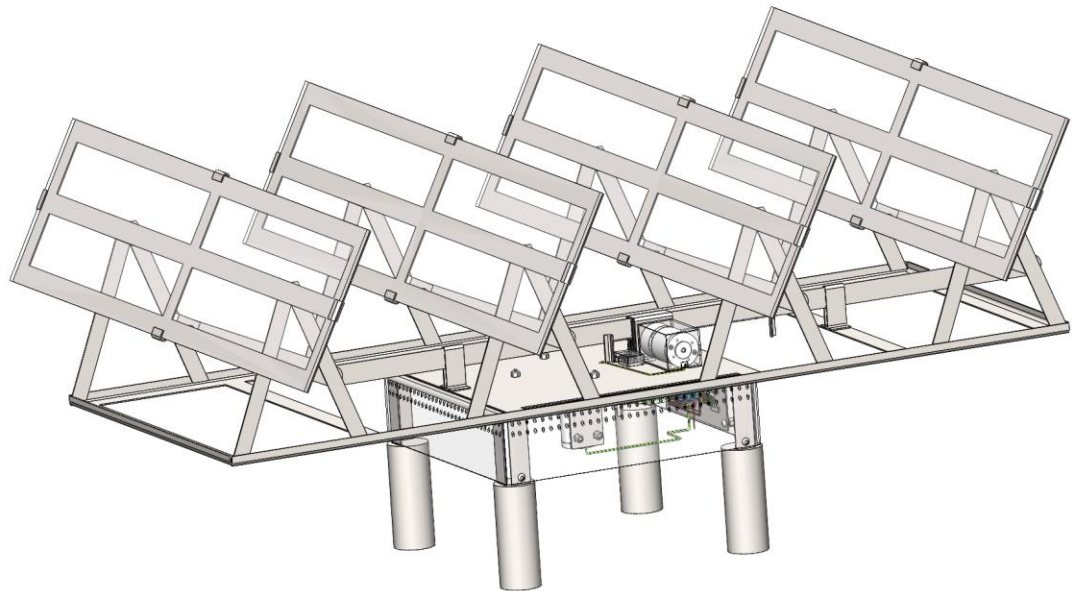
# Our Principals (Hedgehog Concept)

- To create a durable, modular heliostat system with two axes of rotation that is easy to manufacture and easy to install
- We're passionate about creating something new and challenging to push what is possible



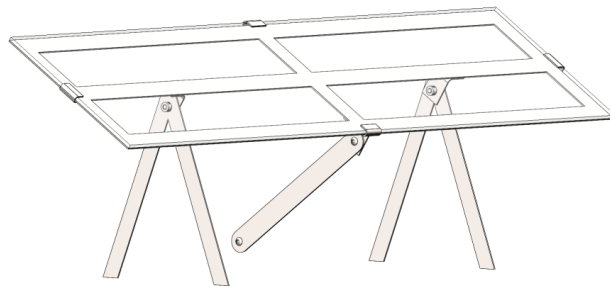
# Design Overview

- Mirror Subsystem
  - 4 reflective surfaces
- Controller subsystem
  - 1 motor
  - Connecting Rod
- Heliostat Base Subsystem
  - Linkage system
  - Rotates about vertical axis
- Module Base Subsystem
  - 1 motor
  - Control Panel
    - Arduino and ESP module
  - 4 legs for support
  - Concrete posts

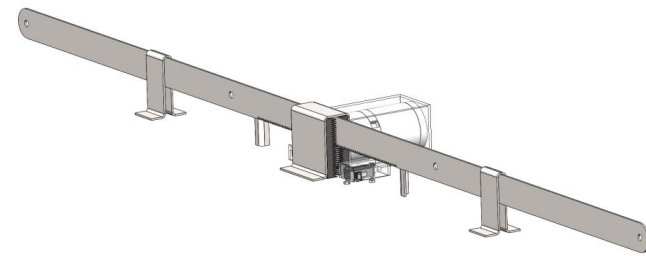


# Design Overview

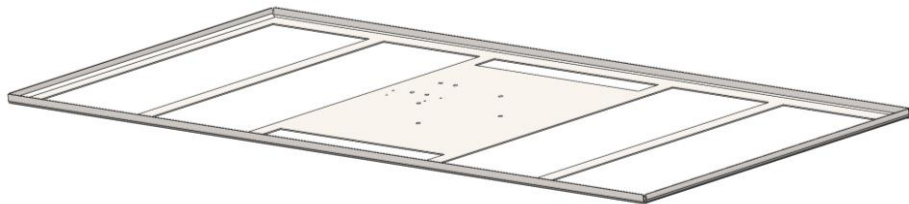
Mirror Subsystem



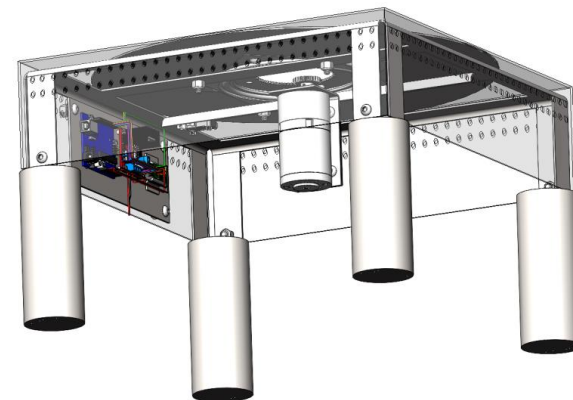
Controller Subsystem



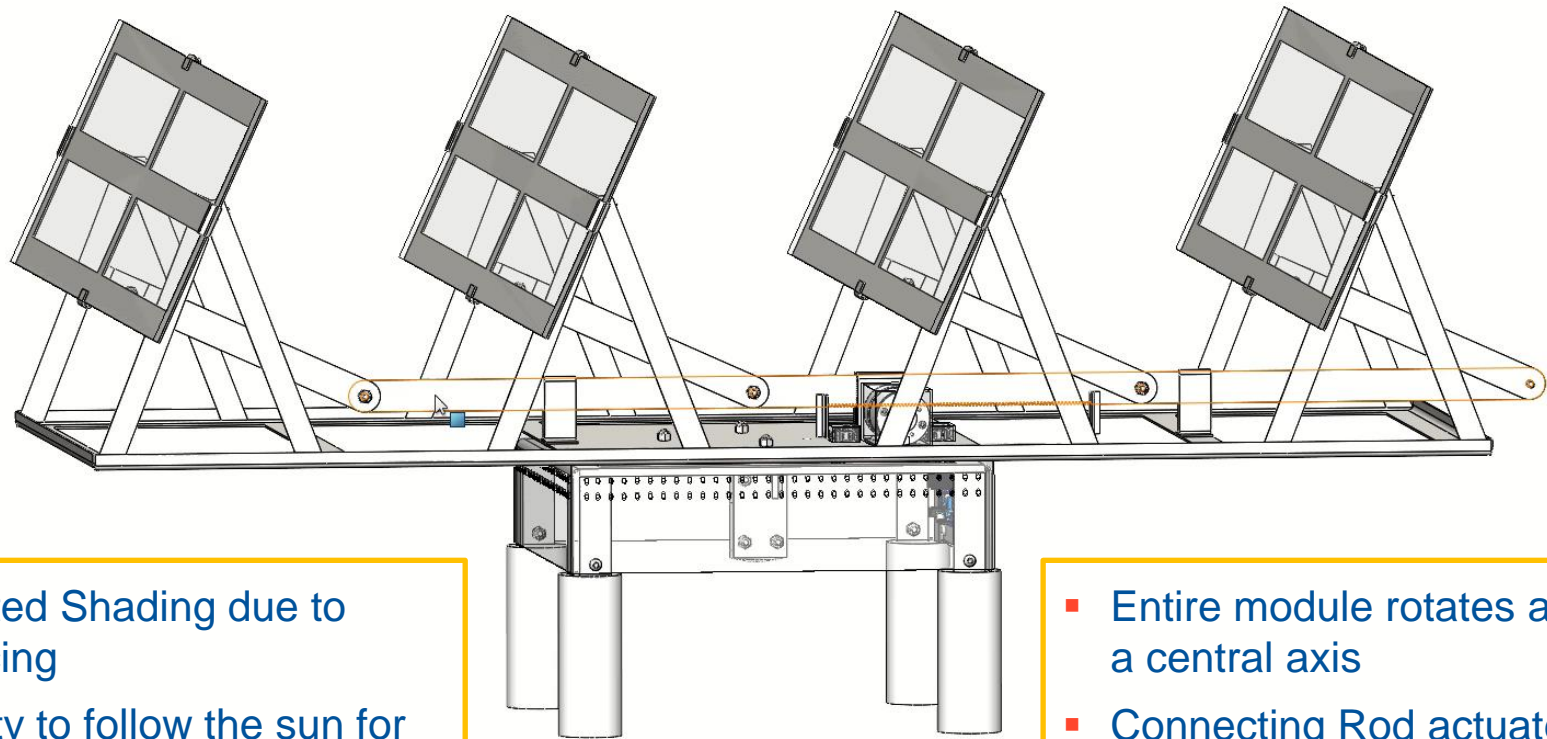
Heliostat Base Subsystem



Module Base Subsystem



# Design Overview

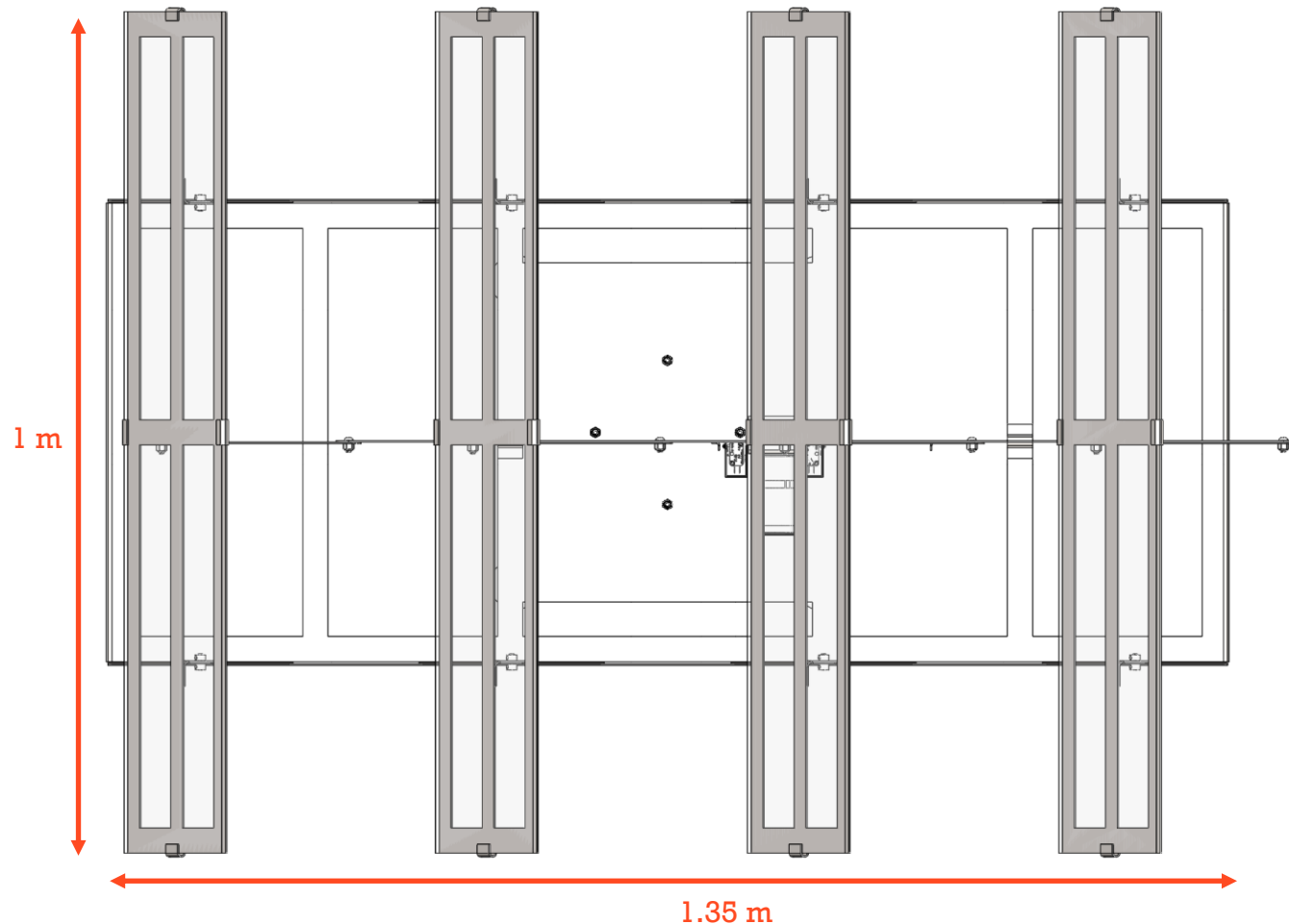


- Limited Shading due to spacing
- Ability to follow the sun for maximum energy collection

- Entire module rotates about a central axis
- Connecting Rod actuates all four mirrors together

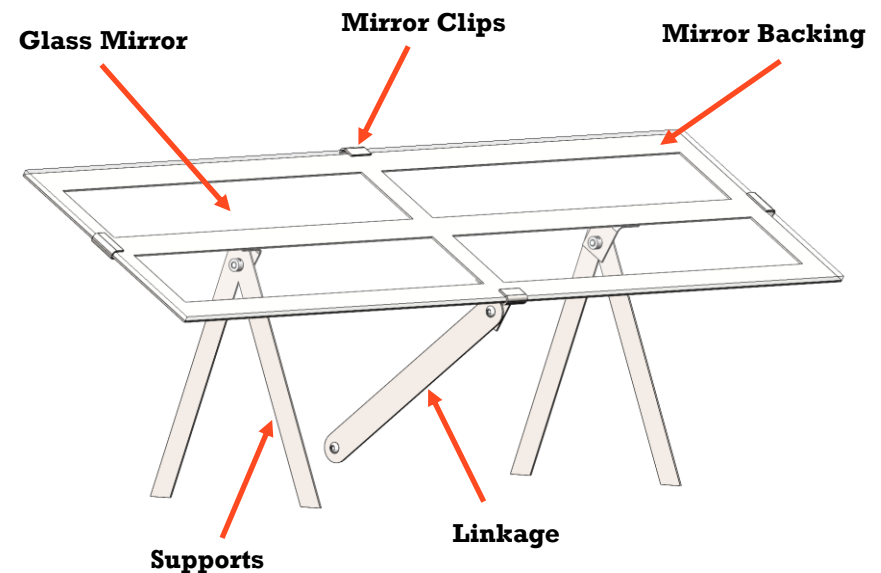
# Overall Product Dimensions

- Top View
- Overall Dimensions
  - 1m x 1.35m
- The center of each module is spaced 1.7m from the nearest module



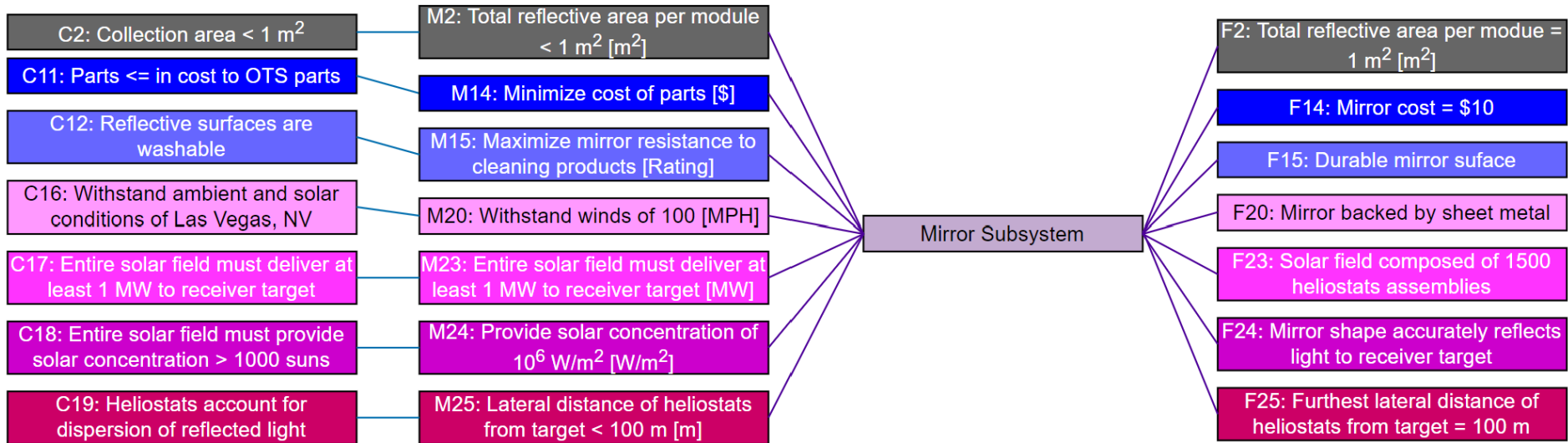
# Mirror Subsystem

- **Borosilicate Glass Mirror**
  - 1 x 0.25 x 0.005 m
- **4 Mirror Clips**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **Mirror Backing**
  - 0.015" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **2 V-Shaped Supports**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **Linkage**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal





# Customer Needs Map for Mirror Subsystem



# Mirror Subsystem Analysis

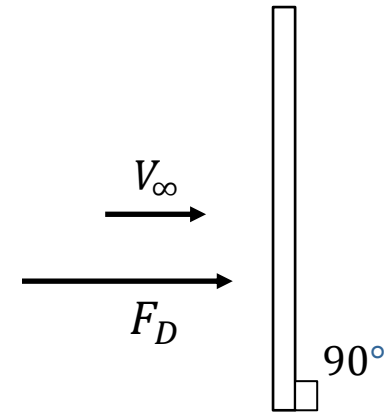
## Drag Force Calculations

- Assume Mirrors are in the vertical position to maximize frontal area

$C_D = 1.98$  (For flat rectangular plate perpendicular to the flow)

$$F_D = \frac{1}{2} \rho v^2 C_D A = \frac{1}{2} \left( 1.225 \frac{\text{kg}}{\text{m}^3} \right) \left( 44.7 \frac{\text{m}}{\text{s}} \right)^2 (1.98) (0.25 \text{ m}^2) = 605.8 \text{ N}$$

$$F_{max} = F_D \times FOS = 605.8 \text{ N} \times 2 = 1211.6 \text{ N}$$



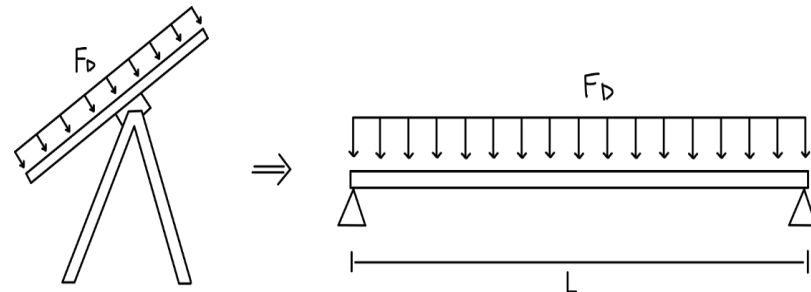
## Mirror Bending Stress Calculations

- Assume Drag Force acts as a distributed load across the face of the mirror

$$M = \frac{F_{max} L^2}{8} = \frac{(1211.6 \frac{\text{N}}{\text{m}}) (1 \text{ m})^2}{8} = 151.45 \text{ Nm}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} (0.25 \text{ m}) (0.00318 \text{ m})^3 = 6.7 \times 10^{-10} \text{ m}^4$$

$$\sigma_f = \frac{M y}{I} = \frac{151.45 \text{ Nm} \times 6.7 \times 0.00159 \text{ m}}{6.7 \times 10^{-10} \text{ m}^4} = 359.44 \text{ MPa}$$



# Mirror Subsystem Analysis

## Downforce Calculation

- Assume mirrors at maximum lifting angle before the flow separates

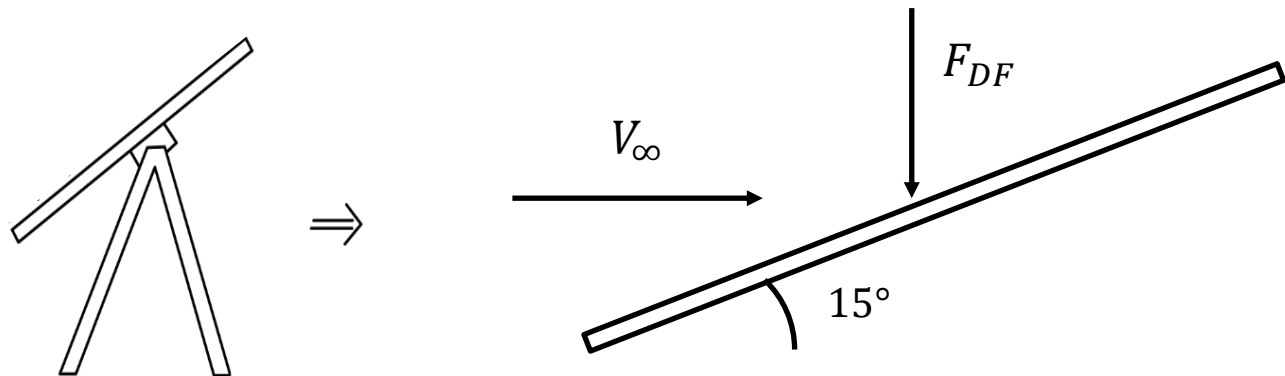
$$C_L = 2\pi\alpha \text{ (For Flat Rectangular Plate)}$$

$$\alpha = 15 \text{ degrees} = 0.2618 \text{ radians (Maximum angle of attack before flow separation)}$$

$$C_L = 2\pi(0.218) = 1.32$$

$$F_{DF} = -F_L = -\frac{1}{2}\rho v^2 C_D A = -\frac{1}{2}\left(1.225 \frac{\text{kg}}{\text{m}^3}\right)\left(44.7 \frac{\text{m}}{\text{s}}\right)^2 (1.32)(0.25 \text{ m}^2) = -402.69 \text{ N}$$

$$F_{max} = F_{DF} \times FOS = 402.69 \text{ N} \times 2 = 805.38 \text{ N}$$



# Maximum Shading

## Assumptions

1. Maximum angular offset from horizontal is  $\theta_s = 40^\circ$
2.  $h_s$  is offset from vertical by the angle  $90^\circ - \theta_s$
3. Mirror position at  $\theta_s$  has tracking errors  $< 0.5^\circ$
4. A right angle is formed between the shaded regions and mirrored surfaces
5. Height of mirrors is 0.25m

Determining height of shaded region:  $h_s$

$$\tan(90^\circ - \theta_s) = \left( \frac{0.25m - h_s}{\text{distance between mirrors}} \right)$$

$$\tan(90^\circ - 40^\circ) = \left( \frac{0.25m - h_s}{0.1575m} \right)$$

$$0.1575m \times \tan(50^\circ) = 0.25m - h_s$$

$$0.25m - (0.1575m \times \tan(50^\circ)) = h_s = \mathbf{0.0623m}$$

Determining area of shaded region per mirror:  $A_{s,mirror}$

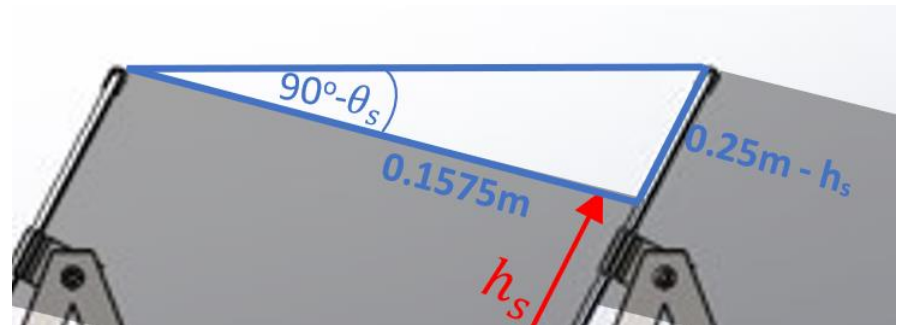
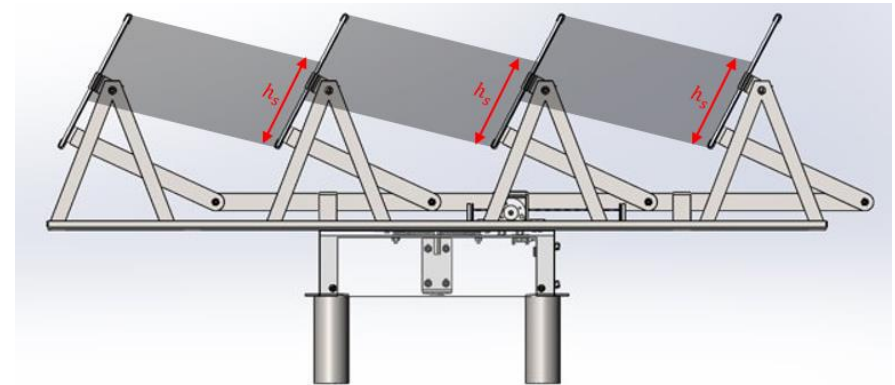
$$A_{s,mirror} = L \times h_s$$

$$A_{s,mirror} = (1m) \times (0.0623m) = 0.0623m^2$$

Determining total shaded area:  $A_{s,total}$

$$A_{s,total} = 3(L \times h_s)$$

$$A_{s,total} = 3(0.0623m^2) = \mathbf{0.187m^2}$$

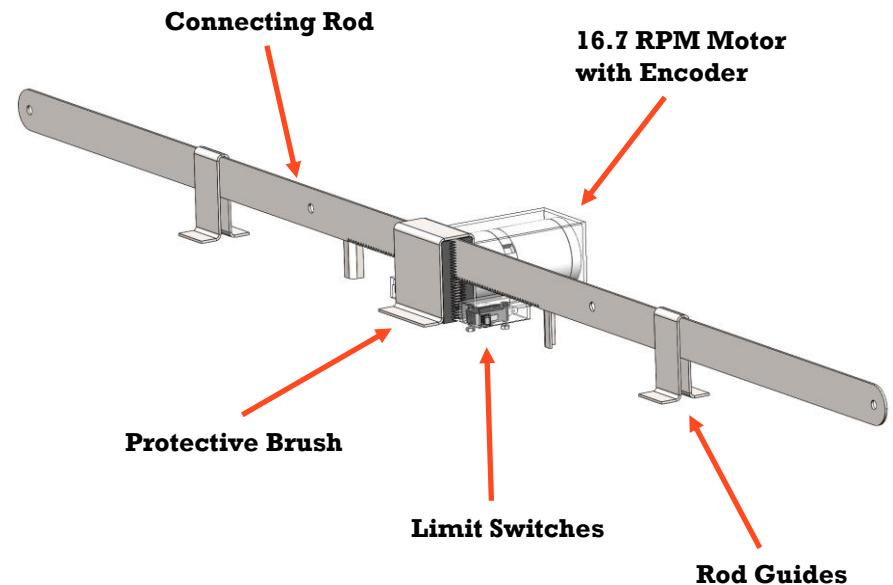


\*Geometry not to scale

$$\%_s = \frac{\text{shaded mirror area}}{\text{total mirror area}} \times 100 = \frac{0.4987m^2}{4(1m \times .25m)} \times 100 = \mathbf{18.7\%}$$

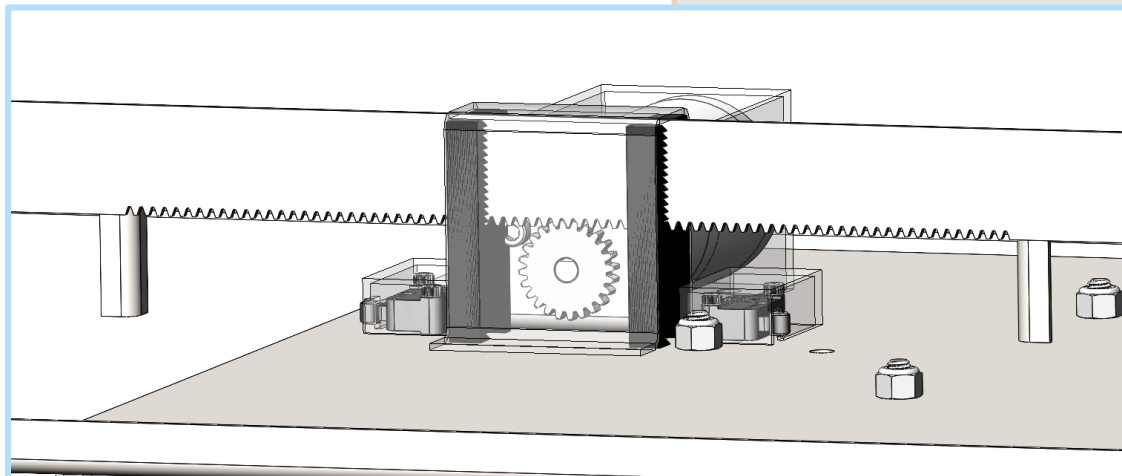
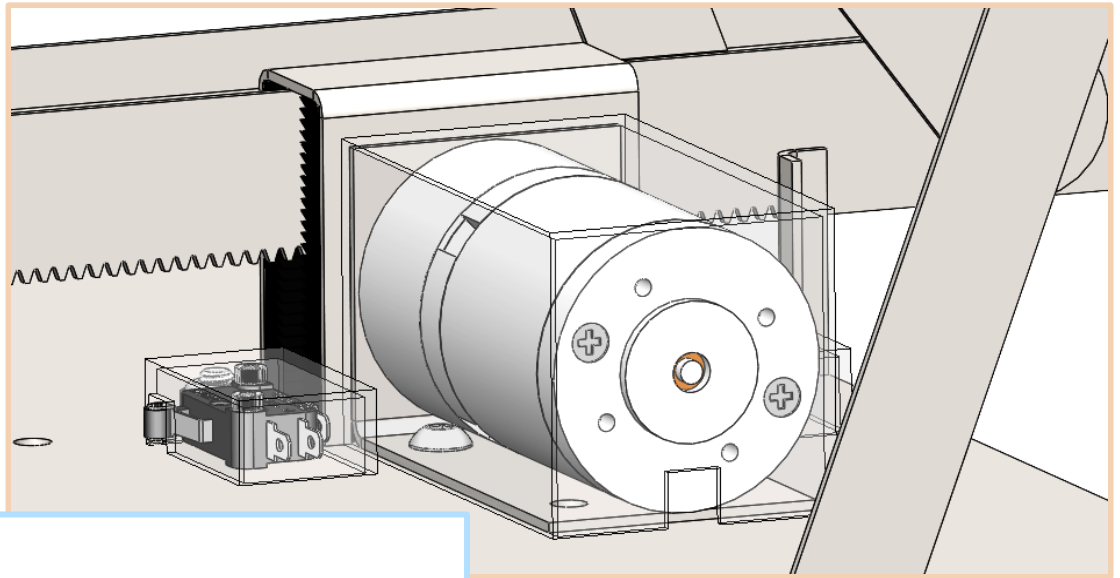
# Controller Subsystem

- **Connecting Rod**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **16.7 RPM Cytron Motor with Encoder**
  - Shielding Case Polyethylene Sheet
  - 810 Counts Per Revolution
- **Protective Brush**
  - Silicone Weather Stripping Brush
  - Brush holder
    - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **2 Limit Switches**
  - Shielding Case Polyethylene Sheet
- **2 Rod Guides**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal



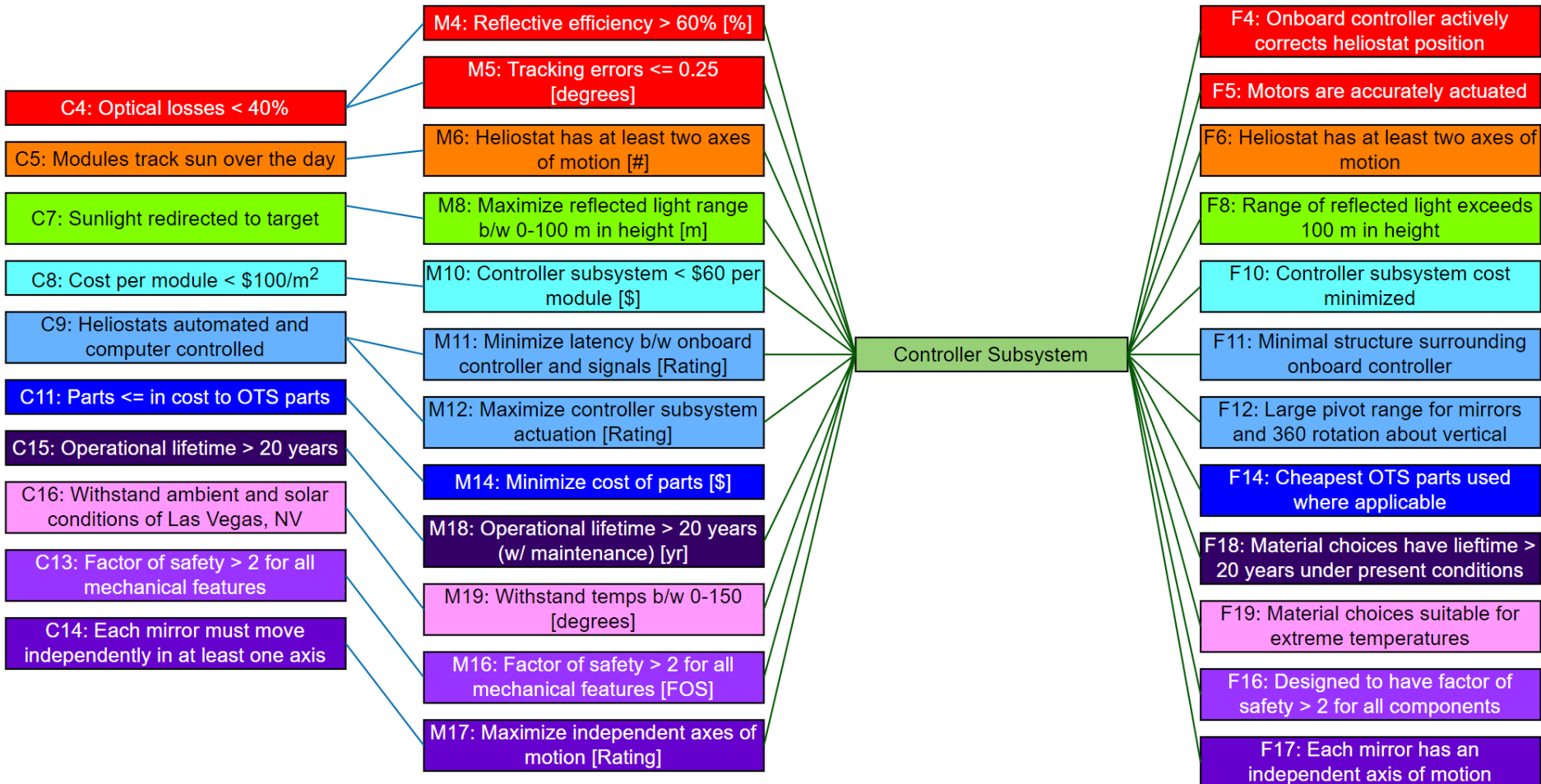
# Controller Subsystem Close Ups

- Motor
- Protective Case
- Brush



- Connecting Rod
- Limits Switches and their Stoppers

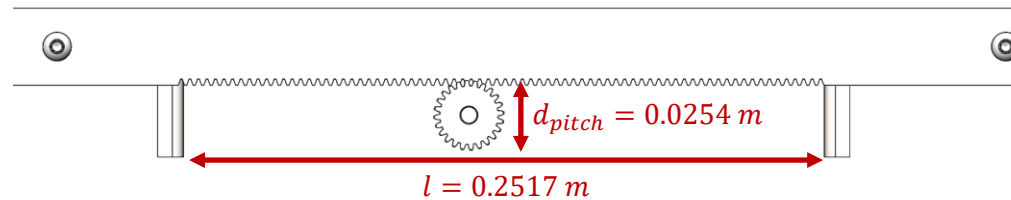
# Customer Needs Map for Controller Subsystem



# Controller Subsystem Analysis

## Motor Torque Calculations

- Assume reset takes 600 seconds, starting from rest



$$v_{rod} = \frac{\Delta l}{t} = \frac{0.2517 \text{ m}}{600 \text{ s}} = 4.195 \times 10^{-4} \text{ m/s}^2$$

$$a_{rod} = \frac{\Delta v_{rod}}{t/2} = \frac{4.195 \times 10^{-4} \text{ m/s}^2}{300 \text{ s}} = 1.398 \times 10^{-6} \text{ m/s}^2$$

$$F_{rod} = m_{rod} a_{rod} = 0.97899 \text{ kg} (1.398 \times 10^{-6} \text{ m/s}^2) = 1.369 \times 10^{-6} \text{ N}$$

$$M = F_{rod} \frac{d_{pitch}}{2} = (1.369 \times 10^{-6} \text{ N}) \left( \frac{0.0254}{2} \text{ m} \right) = 1.739 \times 10^{-8} \text{ Nm}$$

$$\omega = \frac{v_{rod}}{r_{pitch}} = \frac{4.195 \times 10^{-4} \text{ m/s}}{0.0127 \text{ m}} = 0.033 \text{ rad/s}$$

## Power Calculation

$$P_{min} = M\omega = (1.739 \times 10^{-8} \text{ Nm})(0.033 \text{ rad/s}) = 5.265 \times 10^{-7} \text{ W}$$



# Controller Subsystem Analysis

## Heat Transfer Analysis of DC Motor Surface

### Assumptions

1. Surface Thermal Emissivity  $\epsilon = 0.83$
2. Motor  $L = 0.0762$  m, Motor  $D = 0.0508$  m
3. Steady-state system
4. System Efficiency  $\sim 0.8$
5. Air travels through small opening for rack
6. Outer casing heated to  $T_{sur} = 50^\circ\text{C}$  from solar radiation

### Conservation of total energy

$$\frac{dE_{st}^{tot}}{dt} = \dot{E}_{in}^{tot} - \dot{E}_{out}^{tot}$$

### Conservation of thermal/mechanical energy

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

### Relationship between mechanical/electrical power

Mechanical work  $\dot{W} - q - P = 0$  Heat leaving motor

Electrical power

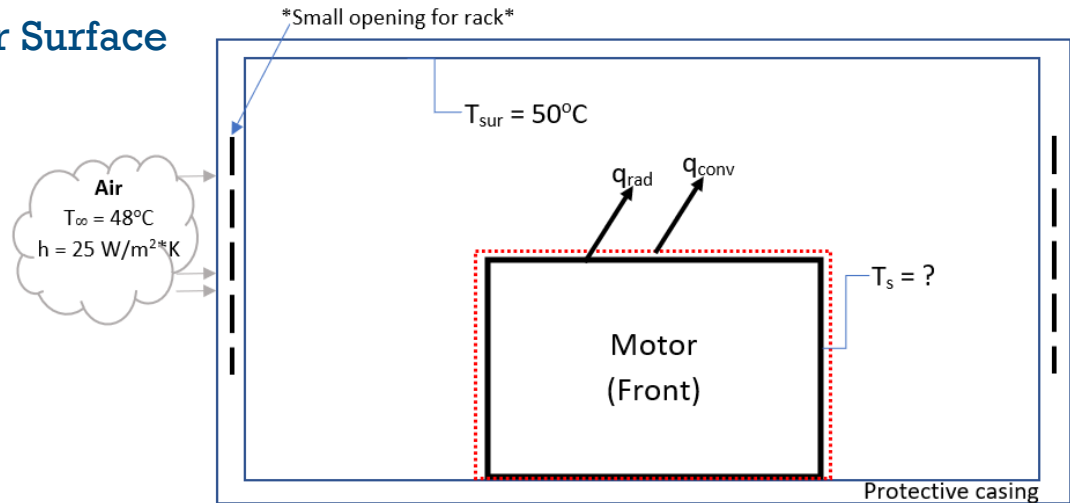
$$q = \dot{W} - P$$

$$P = \dot{W}\eta_{motor}$$

\*Minimum power needed to start movement

$$q = P \left( \frac{1}{\eta_{motor}} - 1 \right)$$

$$q = (1.74 \times 10^{-6} \text{ J}) \left( \frac{1}{0.8} - 1 \right) = 4.35 \times 10^{-7} \text{ W}$$



### Application of Rate Equation

$$q = q_{rad} + q_{conv} = A[q''_{rad} + q''_{conv}]$$

$$q = \left[ \pi DL + \frac{\pi D^2}{4} \right] [\epsilon\sigma(T_s^4 - T_{sur}^4) + h(T_s - T_\infty)]$$

$$4.35 \times 10^{-7} \text{ W} = \left[ \pi(0.0508\text{m})(0.0762\text{m}) + \frac{\pi(0.0508)^2}{4} \right] [\epsilon\sigma(T_s^4 - T_{sur}^4) + h(T_s - T_\infty)]$$

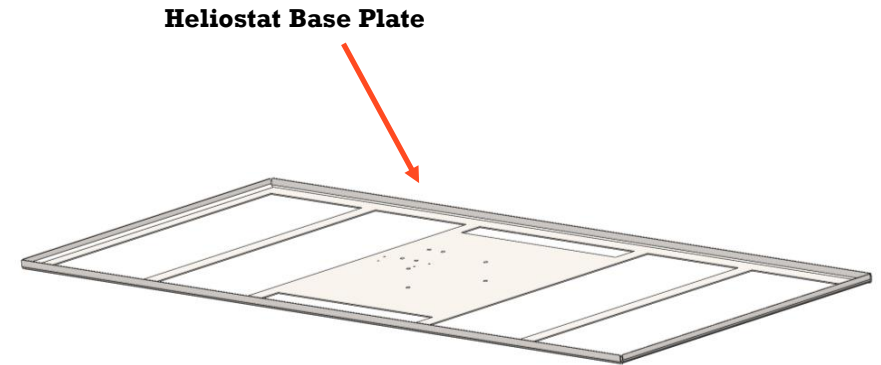
$$3.06 \times 10^{-5} \frac{\text{W}}{\text{m}^2} = \left[ (0.83) \left( 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2} \right) (T_s^4 - 1.088 \times 10^{10} \text{ K}^4) + \left( 25 \frac{\text{W}}{\text{m}^2 \text{ K}} \right) (T_s - 321\text{K}) \right]$$

$$T_s = \text{DC motor surface temp.} = 321.4\text{K} = 48.4^\circ\text{C}$$

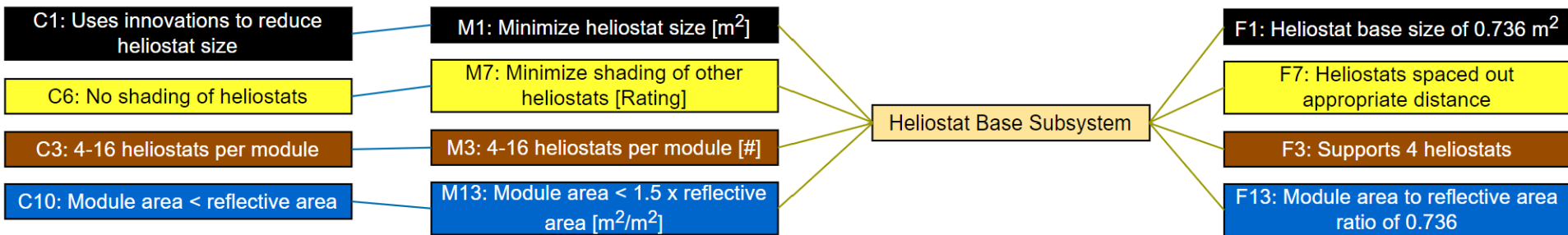
- Component is not at risk of overheating

# Heliostat Base Subsystem

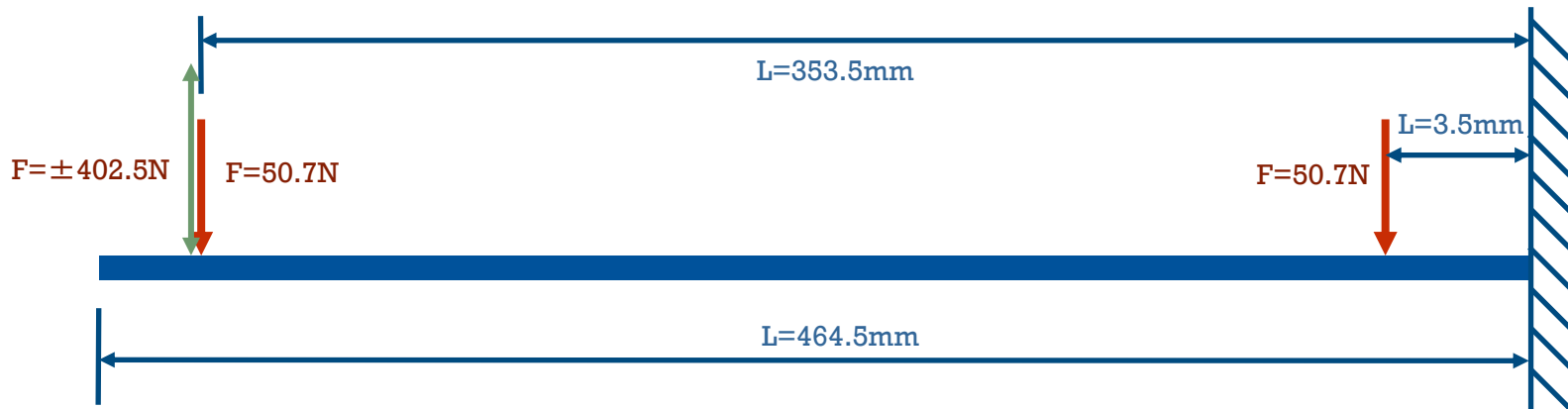
- Heliostat Base Plate
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
  - 1.33 x 0.545 m rectangular shape with cutouts
  - Edges are bent up by 15 mm for strength



# Customer Needs Map for Heliostat Base Subsystem



# Helioostat Base Subsystem Analysis



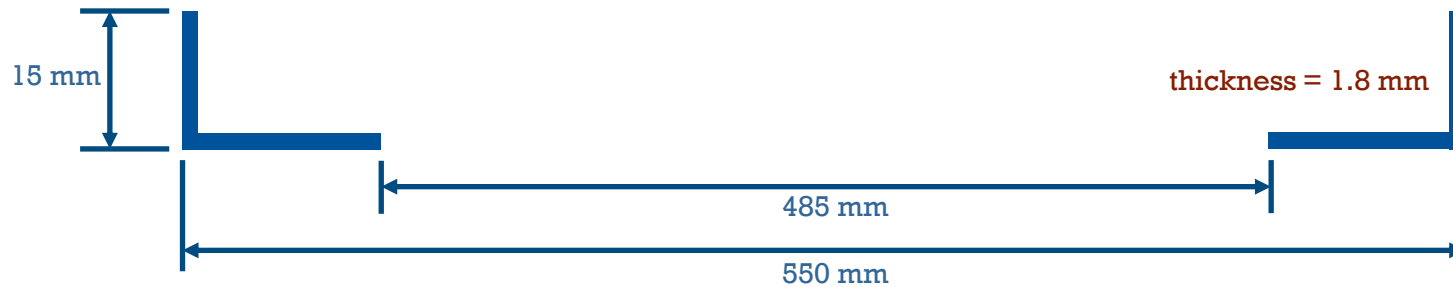
Conservative Cantilever Beam approximation of the helioostat base

$F_1$  and  $F_2$  are forces from the weight of helioostats.  $F_d$  is the max downforce experienced by the mirrors by the wind.

$$M_{max} = \Sigma Fx = F_1x_1 + F_2x_2 + F_dx_d = 160400 \text{ Nmm}$$

$$M_{min} = \Sigma Fx = F_1x_1 + F_2x_2 - F_dx_d = -124200 \text{ Nmm}$$

# Heliostat Base Subsystem Analysis

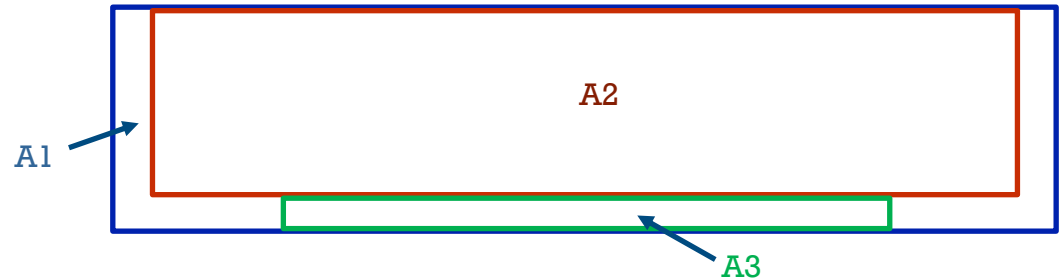


Cross sectional diagram of weakest point

## Centroid calculation

$$\bar{y} = \frac{A_1 y_1 - (A_2 y_2 + A_3 y_3)}{A_1 - (A_2 + A_3)} = 3.066 \text{ mm}$$

$$y = 15 - \bar{y} = 11.93 \text{ mm}$$



## Second Moment of Inertia

$$I = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 - \left( \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 + \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 \right) = 99413.99 \text{ mm}^4$$

## Bending Stresses

$$\sigma = \frac{My}{I}$$

$$\sigma_{max} = \frac{M_{max} y}{I} = 19.26 \text{ MPa}$$

$$\sigma_{min0} = \frac{M_{min} y}{I} = -14.91 \text{ MPa}$$

# Heliostat Base Subsystem Analysis

Cyclic Loading

$$S_{ut} = 635 \text{ MPa}$$

Equivalent Completely Reversed Stress ( $\sigma_{AR}$ )

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 17.08$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 2.17$$

$$\sigma_{AR} = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}} = 17.14$$

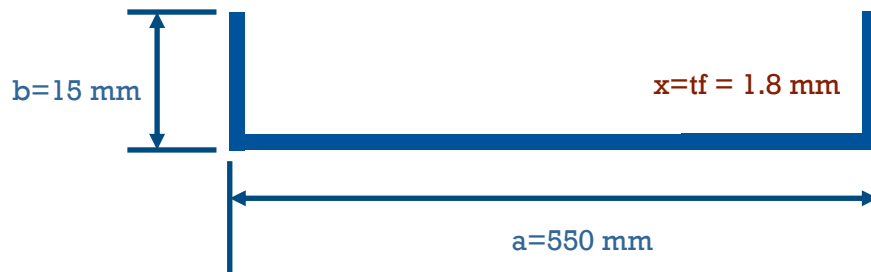


Diagram for equivalent diameter

Marin Factors and Endurance Limit Calculation

$$S'_e = 0.5S_{ut} = 317.5 \text{ MPa}$$

$$k_a = 4.51S_{ut}^{-1.265} = 0.816$$

$$d_e = 0.052xa + 0.1tf(b - x) = 53.9$$

$$k_b = 1.24d_e^{-0.107} = 0.809$$

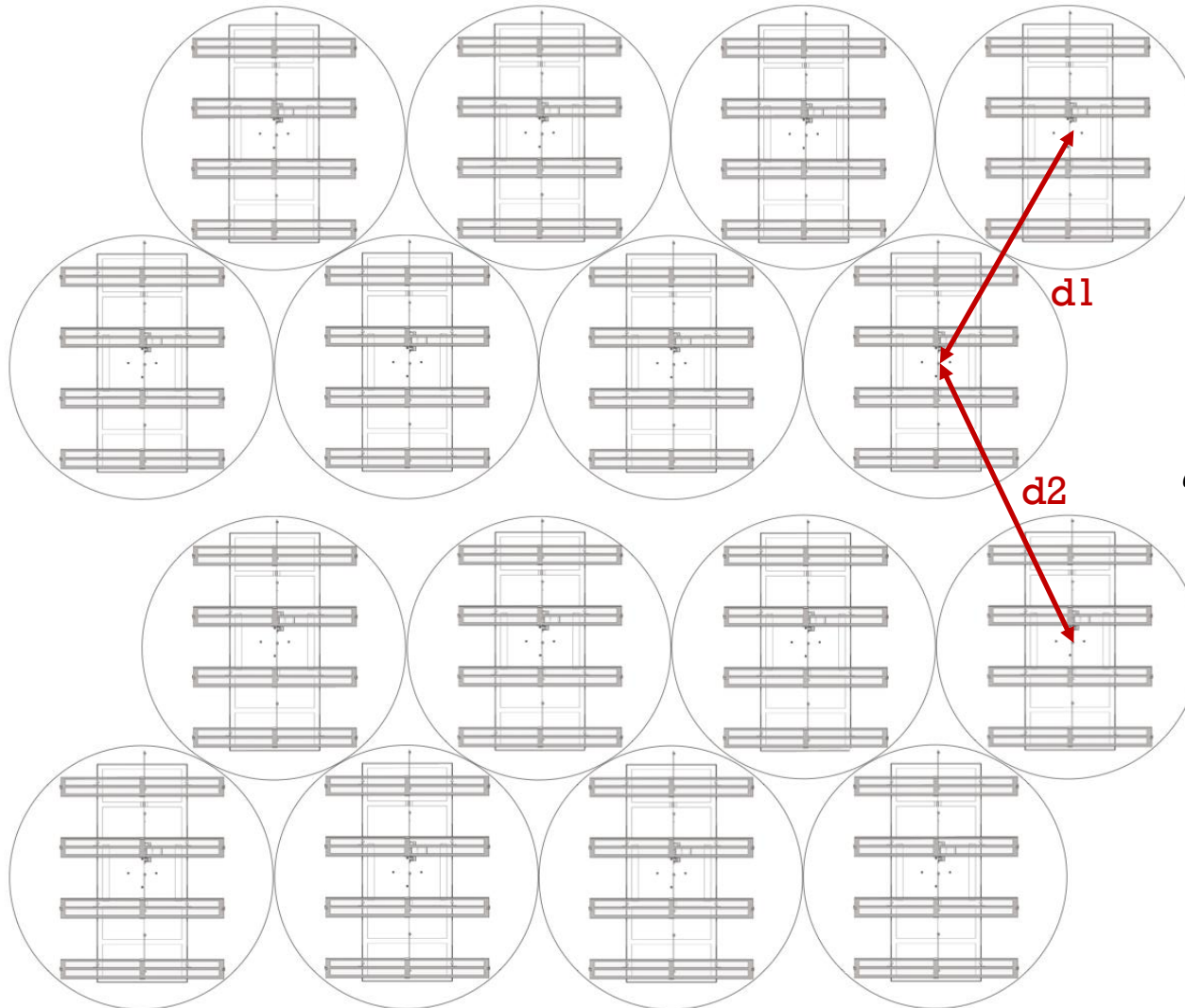
Endurance Limit

$$S_e = S'_e k_a k_b = 209.5 \text{ MPa}$$

Factor of Safety for Infinite Life

$$\eta = \frac{S_e}{\sigma_{AR}} = 12.23$$

# Heliostat Base Subsystem Analysis

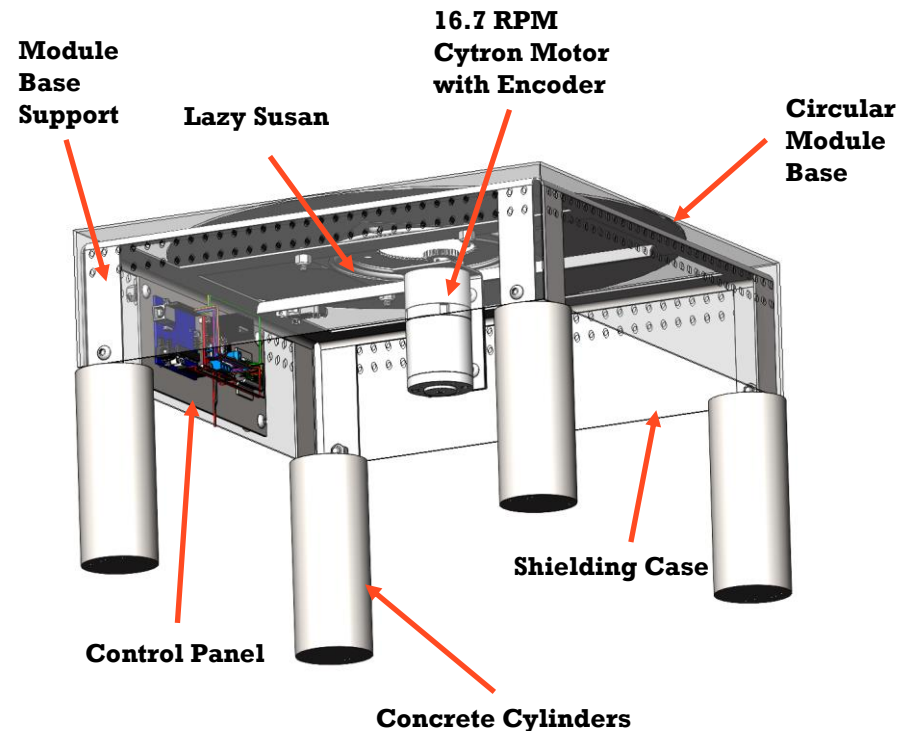


$$d_1 = 1.7m$$

$$d_2 = \sqrt{d_1^2 + \left(\frac{d_1}{2}\right)^2} = 1.9m$$

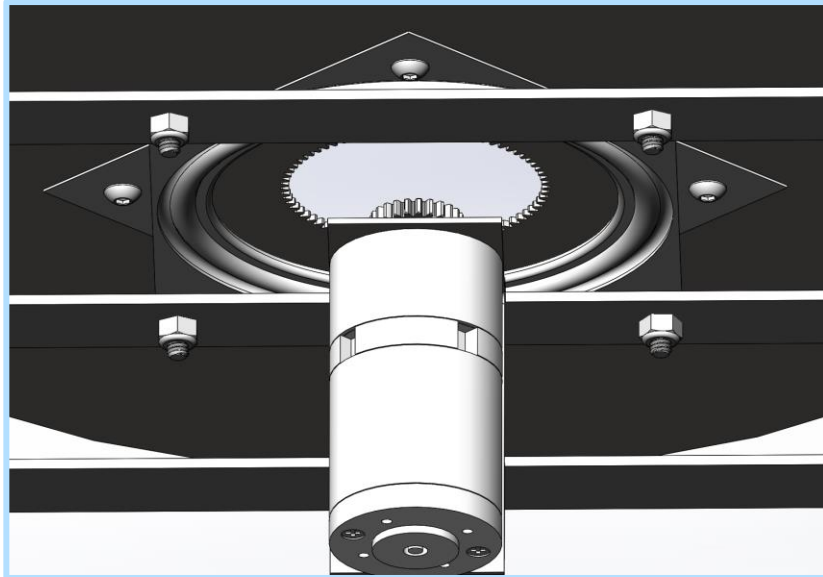
# Module Base Subsystem

- **Module Base Support**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **Lazy Susan**
  - 500lb capacity
- **16.7 RPM Cytron Motor with Encoder**
- **Circular Module Base**
  - 0.071" thick Zinc-Galvanized Low-Carbon Steel Sheet Metal
- **Control Panel**
  - Arduino Uno, 12V motor driver, esp-w2 microcontroller with WiFi capabilities
- **Concrete Cylinders**
- **Shielding Case**
  - Polyethylene Sheet with slits for ventilation

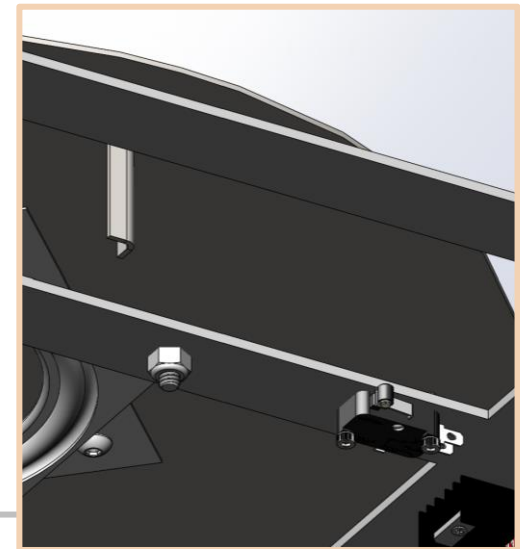




# Module Base Subsystem Close Ups

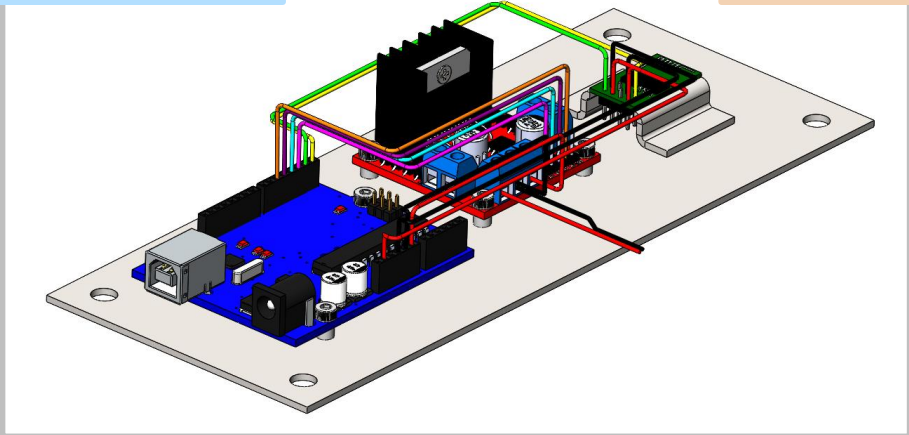


- Motor
- Lazy Susan
- Circular Module Base

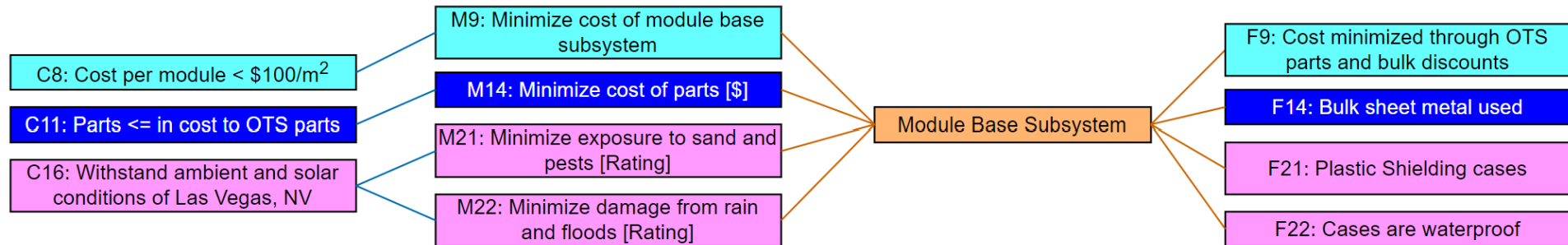


- Limit Switch and its Stopper

- Control Panel**
- Accuracy <math>< 0.5^\circ</math>
  - Wi-Fi Capabilities
  - Encoders to track Position

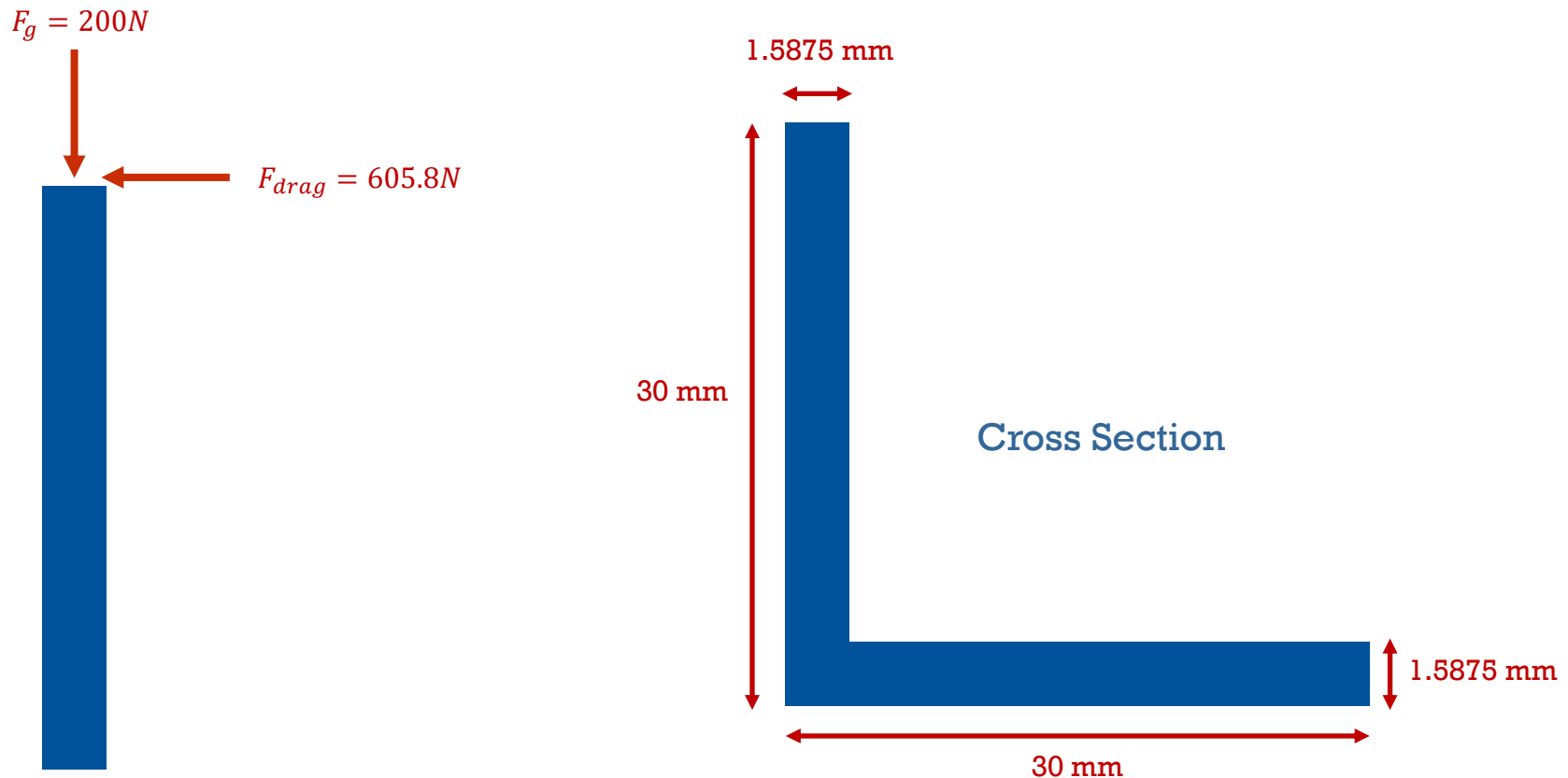


# Customer Needs Map for Module Base Subsystem



# Module Base Subsystem Analysis

## Leg Stress Calculations



# Module Base Subsystem Analysis

## Leg Stress Calculations Cont.

- Normal Stress Calculations

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(1.5875 \text{ mm})(30 \text{ mm})(15 \text{ mm}) + (30 \text{ mm} - 1.5875 \text{ mm})(1.5875 \text{ mm})\left(\frac{1.5875}{2} \text{ mm}\right)}{(1.5875 \text{ mm})(30 \text{ mm}) + (30 \text{ mm} - 1.5875 \text{ mm})(1.5875 \text{ mm})} = 8.0899$$

$$I = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 + \frac{1}{12} b_2 h_2^3 + A_2 d_2^2$$

$$= \frac{1}{12} (1.5875 \text{ mm})(30 \text{ mm})^3 + (30 \text{ mm})(1.5875 \text{ mm})(15 \text{ mm} - 8.0899 \text{ mm})^2 + \frac{1}{12} (30 \text{ mm} - 1.5875 \text{ mm})(1.5875 \text{ mm})^3$$

$$+ (30 \text{ mm} - 1.5875 \text{ mm})(1.5875 \text{ mm}) \left( 8.0899 \text{ mm} - \frac{1.5875}{2} \text{ mm} \right)^2$$

$$= 8256.52 \text{ mm}^4$$

$$\sigma_n = \frac{F_g}{4A} = \frac{200 \text{ N}}{(1.5875 \text{ mm})(30 \text{ mm}) + (30 \text{ mm} - 1.5875 \text{ mm})(1.5875 \text{ mm})} = 2.1568 \text{ Mpa}$$

# Module Base Subsystem Analysis

## Leg Stress Calculations Cont.

- Bending Stress Calculations

$$M = \frac{F_{drag}}{4} h = \frac{605.8 \text{ N}}{4} (100 \text{ mm}) = 15145 \text{ Nmm}$$

$$\sigma_b = \frac{My}{I} = \frac{(15145 \text{ Nmm})(0.79375 \text{ mm})}{8256.52 \text{ mm}^4} = 1.456 \text{ MPa}$$

## Factor of Safety

$$N = \frac{S_{sy}}{\sigma_n + \sigma_b} = \frac{490 \text{ MPa}}{2.1568 \text{ MPa} + 1.4559 \text{ MPa}} = 135.6$$

# Module Base Subsystem Analysis

## Motor Torque Calculations

- Assume reset takes 600 seconds at the end of each day
- Assume the motor starts from rest

$$\frac{N_i}{N_o} = \frac{24 \text{ teeth}}{72 \text{ teeth}} = \frac{1}{3}$$

$$\omega_o = \frac{\Delta\theta}{t} = \frac{2\pi \text{ rad}}{600 \text{ s}} = 0.01047 \text{ rad/s}$$

$$\alpha_o = \frac{\Delta\omega_o}{t/2} = \frac{0.01047 \text{ rad/s}}{300 \text{ s}} = 3.49 \times 10^{-4} \text{ rad/s}^2$$

$$\omega_i = 3\omega_o = 3(0.01047 \text{ rad/s}) = 0.03142 \text{ rad/s}$$

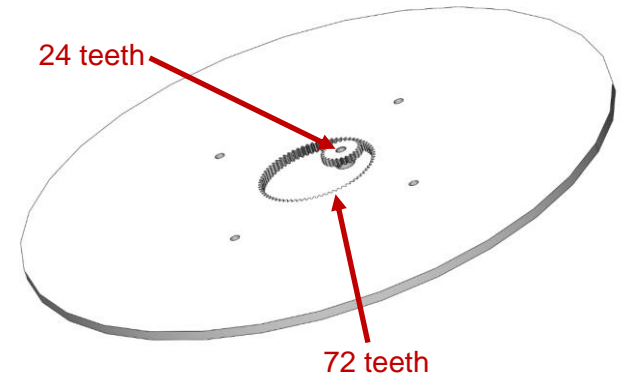
$$\alpha_i = \frac{\Delta\omega_i}{t/2} = \frac{0.03142 \text{ rad/s}}{300 \text{ s}} = 1.05 \times 10^{-4} \text{ rad/s}^2$$

$$M = I\alpha_i = 4.02 \text{ kg} \cdot \text{m}^2 (1.05 \times 10^{-4} \text{ rad/s}^2) = 4.2 \times 10^{-4} \text{ Nm}$$

**\*\*Moment of Inertia I Calculated through SolidWorks\*\***

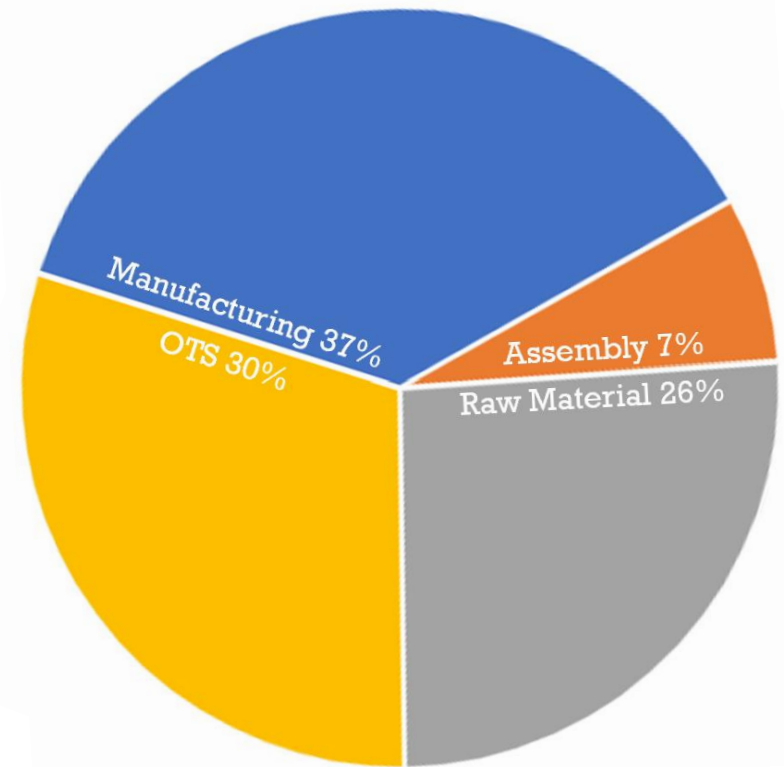
## Power Calculation

$$P_{min} = M\omega_i = 4.2 \times 10^{-4} \text{ Nm} (0.03142 \text{ rad/s}) = 1.32 \times 10^{-2} \text{ W}$$



# Cost Summary – Entire Heliostat Field

Category	Cost	Color
Manufacturing	\$186.53	Blue
Assembly	\$35.72	Orange
Raw Material	\$130.87	Grey
OTS	\$151.68	Yellow
Modified OTS	\$0.00	Light Blue
Energy	\$0.02	Green
<b>Total</b>	<b>\$504.82</b>	



# Lifetime Summary

Although our cost is high, our design's lifespan is predicted to be well over 20 years

Brushless DC Motors have estimated lifespans of 10,000 hours

Our motors will have lifespans of approximately 68.5 years

Borosilicate Glass is highly resistant to thermal stresses

It has an extremely low coefficient of thermal expansion

Most of our design is made from sheet metal parts

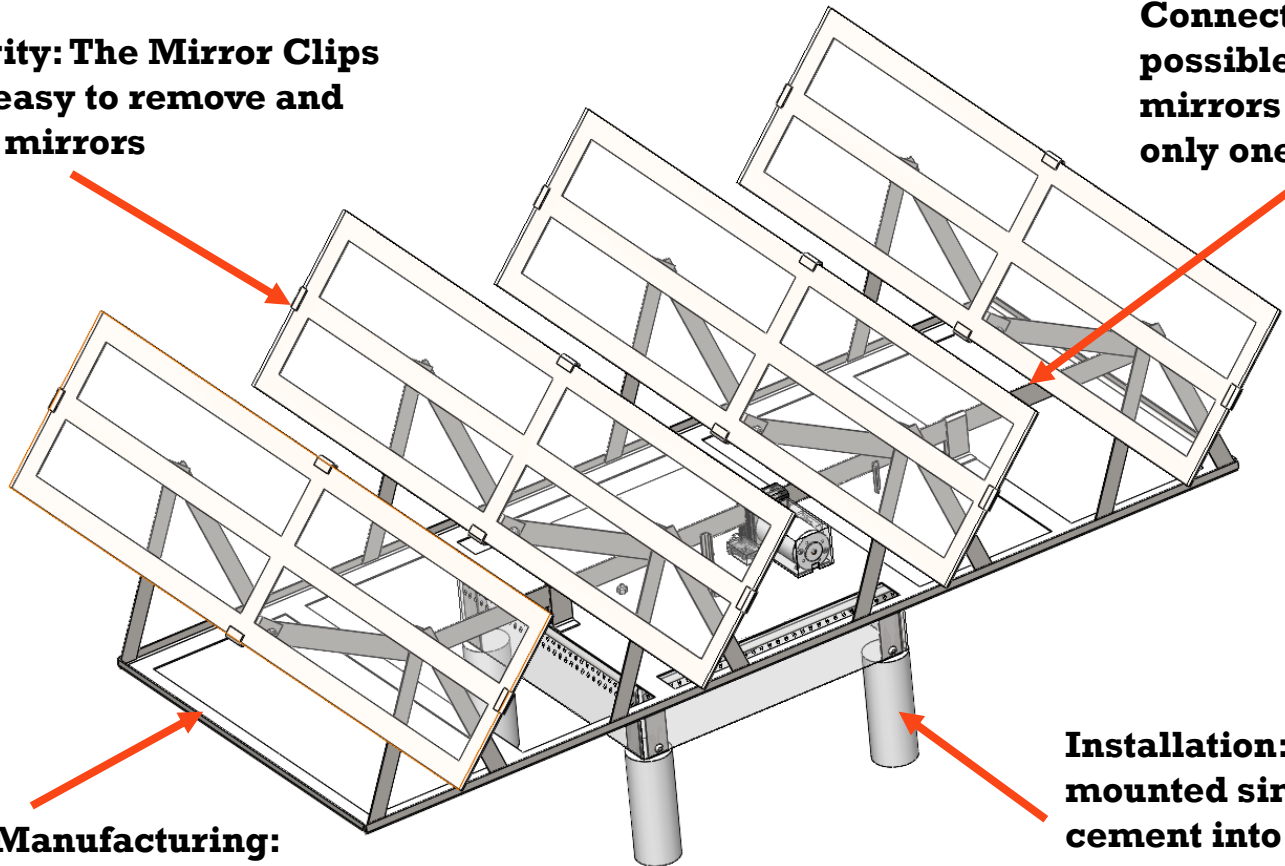
These parts are designed for infinite life, as shown in our analyses



# Design Highlights and Key Features

**Modularity: The Mirror Clips**  
make it easy to remove and  
reattach mirrors

**Minimal Actuation: The  
Connecting Rod** makes it  
possible to rotate four  
mirrors with the use of  
only one motor



**Durability/Manufacturing:**  
Most of the design is made from  
the same type of sheet metal

**Installation: The module is  
mounted simply by pouring  
cement into four holes where  
the module base goes into the  
ground**

# Conclusion

Operation Concentration has designed a product that is

- Easy to manufacture due to the simple materials and techniques
- Quickly installed on the site of the solar farm
- Lower cost than heliostats on the market today
- Modular design to allow for faster maintenance

TM

Thank you to the UF Department of Mechanical and Aerospace Engineering as well as our corporate sponsors!

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Questions?

